LIMITATIONS TO EXTENDED STATIONARY CORONAE

AN ANALYTICAL MODEL

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SUMMARY

A simple analytical model for stellar corona is developed. It is based on the assumption of an isothermal stationary corona and a transition region of constant pressure.
For mathematical convenience it is assumed that the dissipation of mechanical energy takes place in a layer of negligible vertical extend.
It is shown that there is an upper limit for the mechanical flux heating the corona. If this limit is exceeded no extended stationary corona is possible. It is suggested that a thin corona is formed.
The results of the calculations are in good agreement with the numerical results of Hearn and Vardavas (1981).
It is shown that there is a lower bound to the height of the dissipative layer above the chromosphere.

Key words: transition region, stellar coronae, solar coronae.

1. INTRODUCTION

Several authors (Hearn and Vardavas (1981), Hammer (1981), Souffrin (1982) and Hearn et al. (1983)) have found theoretical indications that there exists an upper limit for the mechanical flux heating a corona. Hearn and Vardavas (1981) have suggested that a thin hydrostatic corona is formed if the mechanical flux exceeds this upper limit.

In this paper an analytical for a stellar corona is presented. In this model the energy balance in the transition region is examined. This energy balance is determined by the heating of the corona, the energy losses due to radiation and the divergence of the conducted heat. This balance can be put in the following form

$$\nabla \cdot \underline{J} = q_h - q_r \tag{1}$$

where q_h is the mechanical heating of the corona, q_r is the radiation loss and $\underline{J}$ is the conducted heat. (See Martens (1981))

For a given heating function there is only one solution of (1) consistent with the boundary condition at infinity.

(1) will be solved using heating functions in a mathematical convenient form. The heating function is specified by two parameters. The aim of this work is to find out which combinations of parameters yield extended stationary coronae.

To compare the results with the results of Hearn and Vardavas (1981) the calculations will be done for an OB- supergiant.

It will be confirmed that there is a maximum mechanical flux consistent with an extended stationary corona, and according to the numerical result of Hearn and Vardavas (1981) a solution for a thin hydrostatic corona is found.

Finally the theory will be applied to the solar corona.

2. THE CORONAL MODEL

2.1 The energy equation

This section describes the energy equation which is valid in the transition region. The following assumptions are being made: (Martens 1981)

- The thickness of the transition region is small compared to the stellar radius, so a plane parallel geometry may be used.
- The thickness of the transition region is smaller than the pressure scale height, so constant gas pressure is assumed.
- The stellar wind velocity and energy are negligible.
- The atmosphere consists of fully ionized hydrogen.
- The transition is homogeneous in the horizontal direction, so the model can be one dimensional.

With the assumptions above the gas law can be written as

$$P_o = 2 n_e kT \tag{2}$$

where P_o is the constant gas pressure, n_e the electron density and T is the temperature. k is Boltzmann's constant.
The radiation losses of a hot ionized, optically thin gas of stellar composition can be described by

$$q_r = n_e^2 j(T) \tag{3}$$

(Cox and Tucker 1969, McWhirter et al. 1975)
For $1.5\ 10^4\ K < T < 10^6\ K$ $j(T)$ can be approximated within a factor two by

$$j_o(T) = 1.8\ 10^{-22} \quad (\mathrm{erg\ cm^3\ s^{-1}}) \tag{4}$$

The divergence of the conducted heat is given by

$$\nabla.J = - \frac{d}{dh} \left(\kappa_o T^{5/2} \frac{dT}{dh} \right) \tag{5}$$

where h is the height in the transition region.
An estimate for κ_o is given by Athay (1971, p. 36)

$$\kappa_o = 1.1\ 10^{-6} \quad (\mathrm{erg\ cm^{-1}\ s^{-1}\ K^{-7/2}}) \tag{6}$$

In the calculations the heating function q_h will be a nonnegative function, which is zero beyond some height h_o. The function satisfies

$$\int_0^{h_o} q_h(h)\, dh = F_o \tag{7}$$

where F_o is the total mechanical flux entering the transition region. The following heating function will be used in this paper

$$q_h(h) = F_o\, \delta(\, h - L_o\,) \tag{8}$$

where $\delta(x)$ is the Dirac delta function.
This heating function represents very intens heating at one place L_o. This will be called δ-heating. Physically this means that the mechanical flux is dissipated in a very thin layer around L_o.
Combining (2), (3), (5) and (8) yields

$$\kappa_o \frac{d}{dh} (\, T^{5/2} \frac{dT}{dh}\,) = \frac{j_o\, P_o^2}{4\, k^2\, T^2} - F_o\, \delta(\, h - L_o\,) \tag{9}$$

This is a second order, non linear differential equation for the temperature. The boundary conditions are $T(0) = T_{chrom}$ and $\frac{dT}{dh}(0) = 0$. The last condition means that there is no thermal conduction down into the chromosphere.

In some cases another heating function will be used; the socalled constant heating (Martens 1981)

$$q_h(h) = \frac{F_o}{L_o}\, H(\, L_o - h\,) \tag{10}$$

where $H(x)$ is the Heaviside stepfunction.

2.2 Solution of the energy equation in the case of δ-heating

Equation (9) can be made dimensionless by choosing new variables and y.

$$\eta = (\, T\, /\, T_a\,)^{7/2} \tag{11}$$

$$y = h\, /\, \varepsilon L_o \tag{12}$$

T_a is a reference temperature defined by

$$\frac{j_o\, P_o^2}{4\, k^2\, T_a^2} = \frac{F_o}{L_o} \tag{13}$$

and ε is given by

$$7\, \varepsilon^2 = \kappa_o\, T_a^{7/2}\, /\, L_o F_o \tag{14}$$

The result is

$$2\frac{d^2\eta}{dy^2} = \eta^{-4/7} - \delta(\varepsilon y - 1) \qquad (15)$$

The boundary conditions become $\eta(0)$ 1 and $\frac{d\eta}{dy}(0) = 0$.
By integration of (15) one easily sees that

$$\lim_{y \uparrow \varepsilon} \frac{d\eta}{dy} - \lim_{y \downarrow \varepsilon} \frac{d\eta}{dy} = 1 / 2\varepsilon \qquad (16)$$

This jump in the derivative of η is due to the mathematical properties of the Dirac delta function, which are not of physical interest, since the heating will take place in some finite interval.
For $y < 1/\varepsilon$ and $\eta(0) = 0$ the solution of (15) is

$$\eta(y) = \left(\frac{11}{2\sqrt{21}} \, y \right)^{14/11} \qquad (17)$$

Because $\eta(1/\varepsilon)$ has to represent a temperature maximum ε has to be less than 0.201. This follows from (16) and (17). A physical expanation for this is that if ε exceeds 0.201 all the dissipated mechanical energy is conducted into the transition region, where it is radiated away. This means there is no enrgy supply to the corona.
For $y > 1/\varepsilon$ the first integral of (15) is

$$\left(\frac{d\eta}{dy}\right)^2 = \frac{7}{3}\eta^{3/7} + c \qquad (18)$$

where c is an integration constant.
The condition imposed by (16) and the continuity of η give

$$c = \frac{1}{4\varepsilon^2} - \frac{7}{3}\left(\frac{11}{2\sqrt{21}}\right)^{3/11} \varepsilon^{-14/11} \qquad (19)$$

One now can distinguish three cases: $c > 0$, $c < 0$ and $c = 0$.

1. $c > 0$, $\varepsilon < 0.078$
   this gives

$$\frac{d\eta}{dy} = -\left(\frac{7}{3}\eta^{3/7} + c\right)^{\frac{1}{2}} < -\, c^{\frac{1}{2}} \qquad (20)$$

   This is not an acceptable solution, because η falls down very rapidly and becomes less than zero.

2. $c < 0$, $0.078 < \varepsilon < 0.201$ (See fig. 1)

   In this case the region beyond the temperature maximum can be approximated fairly well as isothermal. One finds a solar type corona with a modest massloss. Martens (1981) has calculated coronal models using expressions for the energy losses that were derived by Hearn (1975).

   The energy flux entering the corona is

$$F = F_o - \int_0^{L_o} q_r(h)\, dh = (1 - 3.21\, \varepsilon^{8/11})\, F_o \qquad (21)$$

   This will be further discussed in section 3.

3. $c = 0$, $\varepsilon = 0.078$ (See fig. 1)

   The exact solution of (15) is for $1/\varepsilon < y < 2/\varepsilon$

$$\eta(y) = (\eta_{max}^{11/14} - \frac{11}{2\sqrt{21}} (y - 1/\varepsilon))^{14/11} \qquad (22)$$

   In this case one can calculate that

$$\int_0^{2L_o} q_r(h)\, dh = F_o \qquad (23)$$

   so all the dissipated energy is radiated away. The solution representsa thin corona. Beyond $h = 2\, L_o$ the atmosphere will be in radiative equilibrium, since no additional heating is present.

Finally, in the transition region the follwing relations can be derived for the gas pressure and the maximum temperature

$$P_o = 2\, k\, j_o^{-\frac{1}{2}} (7/\kappa_o)^{2/7}\, L_o^{-3/14}\, F_o^{11/14}\, \varepsilon^{4/7} \qquad (24)$$

$$T_{max} = (11/2\sqrt{21})^{4/11}\, (7/\kappa_o)^{2/7}\, (L_o\, F_o)^{2/7}\, \varepsilon^{16/77} \qquad (25)$$

These expressions will be used in section 3.

2.3 The possibility of a thin corona

The case $c = 0$ can be interpreted as a thin corona. Numerical calculations of Hearn and Vardavas (1981) and Vardavas and Hearn (1981) have been done for stationary stellar coronae. The calculations concerned an OB supergiant with an effective temperature of 31000 K, a mass of 44.7 solar masses and a radius of 27.8 solar radii. The effect of electronscattering radiation pressure was included by reducing the effective stellar mass to 25.108 solar masses.

These coronae were heated by sawtooth waves with a period of 17000 seconds. The mechanical flux was specified deep in the photosphere. Fluxes of 10^3, 10^4 and 10^5 erg cm^{-2} s^{-1} yielded extended stationary coronae. However, with a flux of 10^6 erg cm^{-2} s^{-1} the densities in the outer regions of the corona became so high that thermal conduction could no longer maintain the high temperatures. The corona collapsed and a thin hydrostatic corona was formed. This corona had a thickness of only 0.03 stellar radius.

The results of Vardavas and Hearn (1981) and of the calculations done in section 2.2 are shown in table 1.

	δ- heating	constant heating	Vardavas and Hearn (1981)
P_o (dyne cm^{-2})	$7.1\ 10^{-3}$	$6.3\ 10^{-3}$	$6.71\ 10^{-3}$
T_{max} (K)	$9.8\ 10^5$	$7.0\ 10^5$	$6.98\ 10^5$

TABLE 1. Comparison with the calculations of Vardavas and Hearn (1981). The pressure and maximum temperature of a thin corona of an OB supergiant. The mechanical flux entering the transition region is $2.5\ 10^4$ erg cm^{-2}s^{-1}. The thickness of the corona is $0.03\ R = 5.8\ 10^{10}$ cm. The solution achieved for constant heating is the one that satisfies $T(0) = T(L_o)$.

From table 1 it is clear that the analytical results are in good agreement with the numerical result of Vardavas and Hearn.

The effect of the type of heating function on the coronal pressure is not very important. Only the temperature varies rather strongly. The reason one finds a maximum temperature that is higher in the case of δ-heating is that the dissipation of mechanical enrgy takes place in a very thin layer.

The question that arises is: when is an isothermal model for a corona applicable and when is a thin corona possible? In other words: what are the limitations to extended stationary coronae?

This question will be answered in the next section.